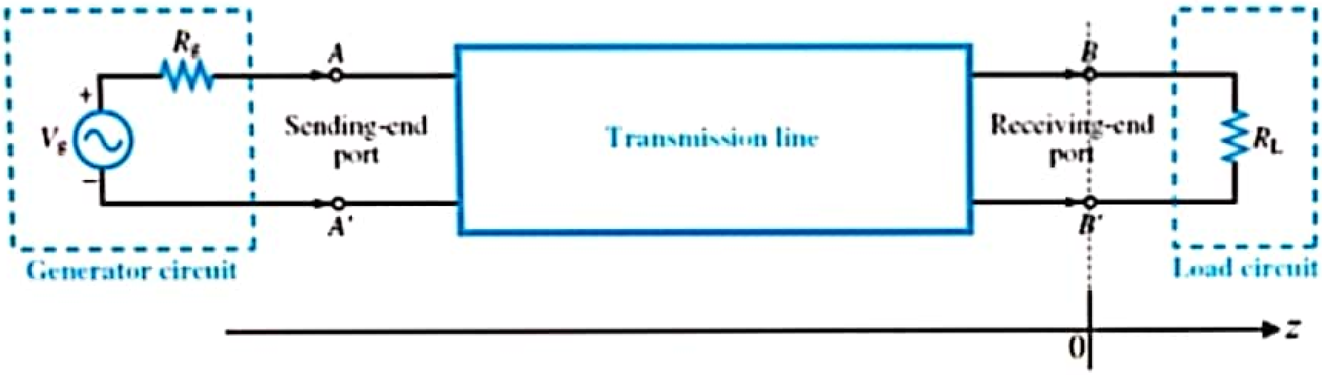
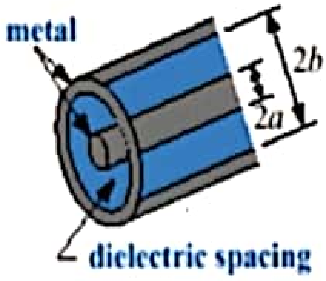
Transmission Line

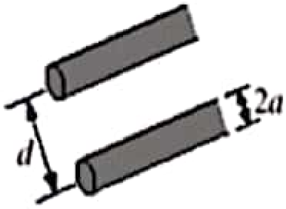


A transmission line is a two-port network connecting a generator circuit at the sending end to a load at the receiving end.

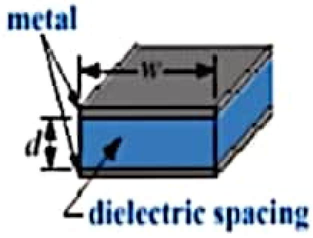
Common Types of Transmission Lines



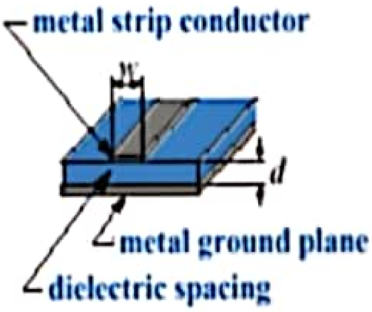
(a) Coaxial line



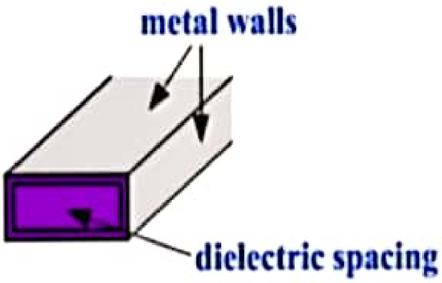
(b) Two-wire line



(c) Parallel-plate line



(d) Microstrip line



(e) Waveguide

Examples of Transmission Line

Transmission Line in communication carry

1)Telephone signals

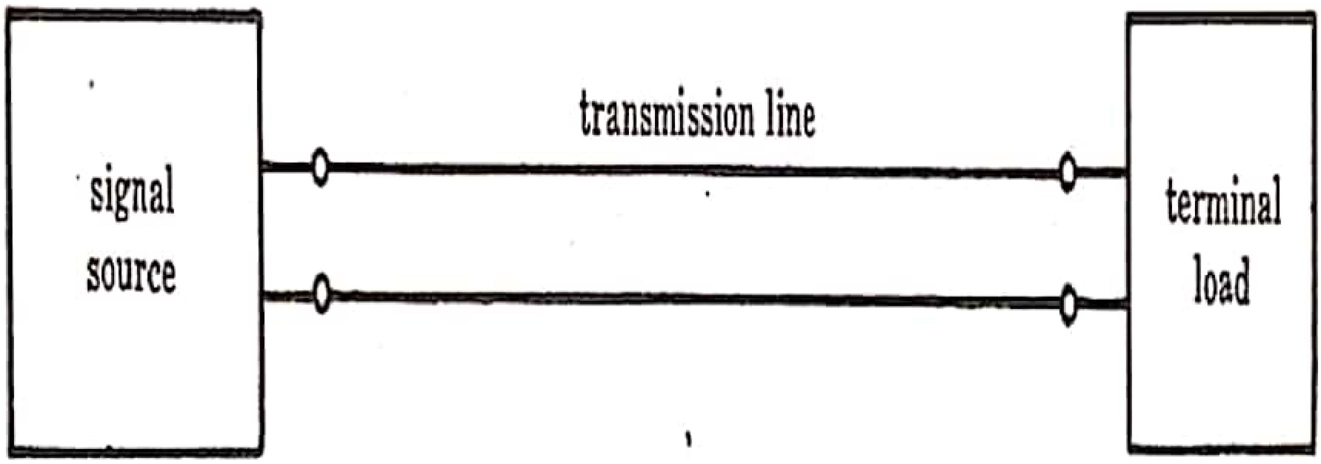
2)Computer data in LAN

3)TV signals in cable TV network

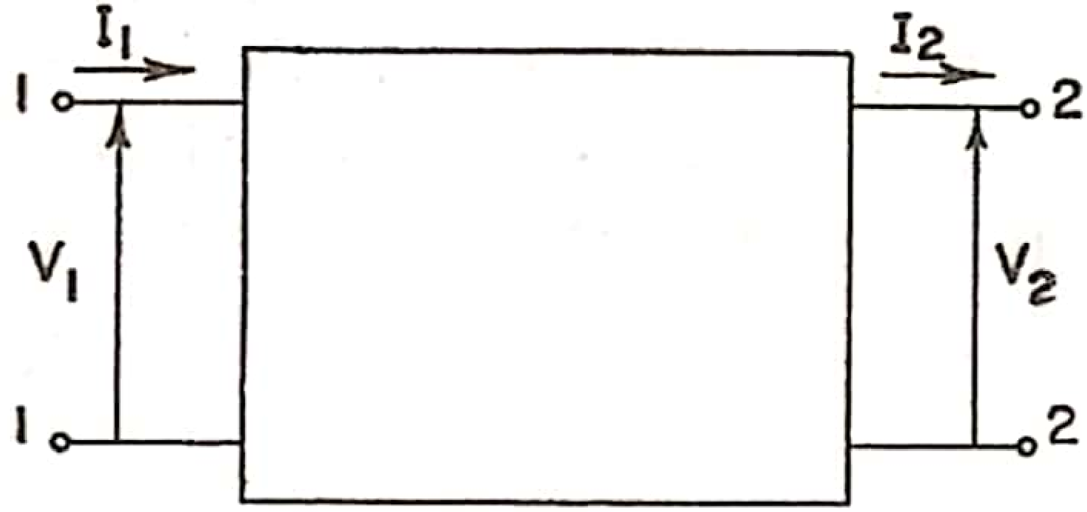
4)Telegraph signals

5)Antenna to transmitter link

Basic Transmission Line

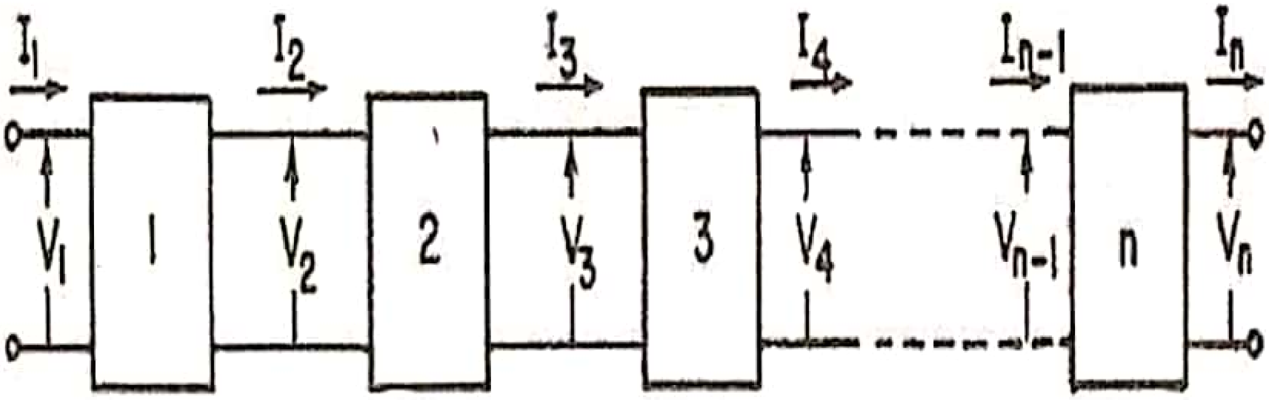


Any complicated network with terminal voltage and current indicated

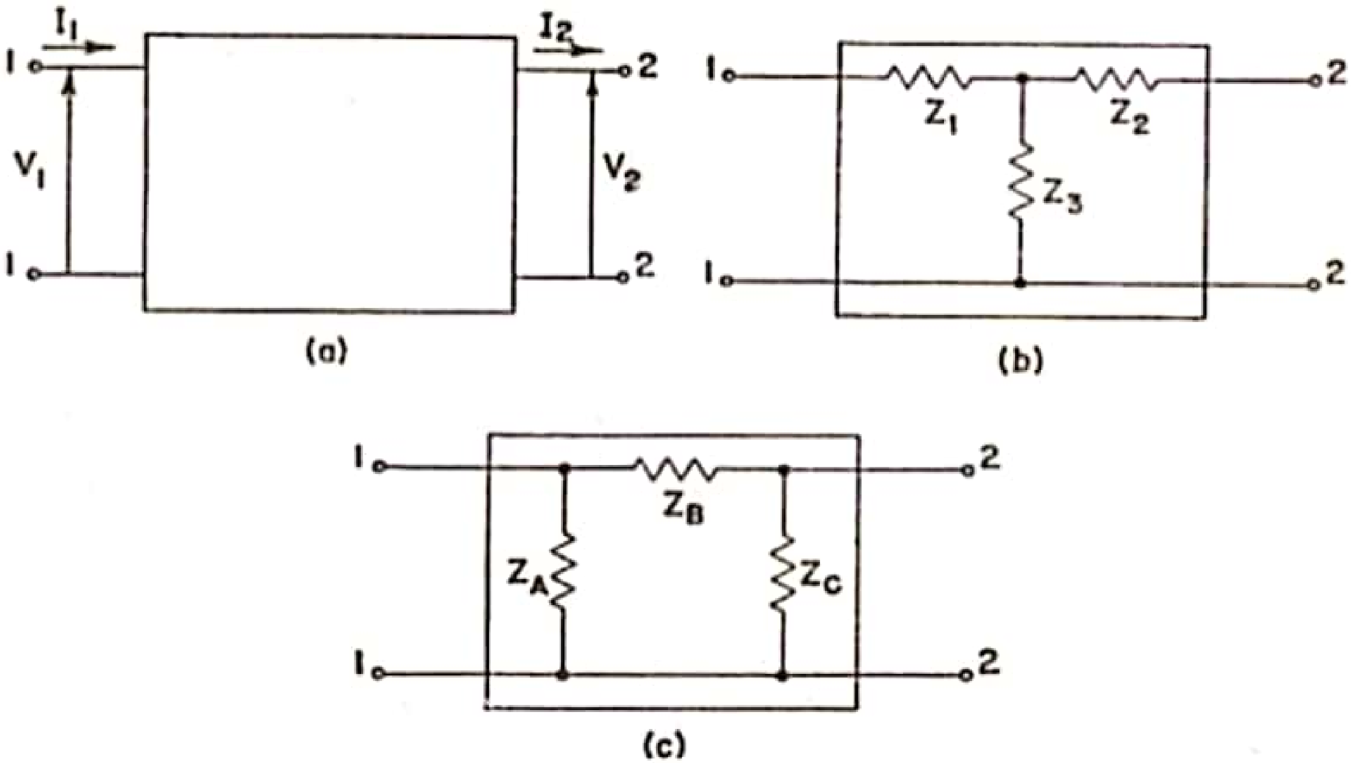


(a)

A succession of n networks in cascade.



Any complicated network can be reduced to T or π network



Distributed Parameters

We use the following distributed parameters to characterize the circuit properties of a transmission line.

R' = resistance per unit length, **(Ω/m)**
L' = inductance per unit length, **(H/m)**
G' = conductance per unit length, **(S/m)**
C' = capacitance per unit length, **(F/m)**
Δz = increment of length, **(m)**

Characteristic impedance, Z_o

- Characteristic impedance, Z_o
  - Ratio of voltage to current, $V/I = Z$
  - From transmission line model

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Propagation constant γ

$$\left|\frac{V_1}{V_2}\right| = \left|\frac{I_1}{I_2}\right| = e^N$$

- The magnitude ratio does not express the complete network performance , the phase angle between the currents being needed as well.
- The use of exponential can be extended to include the phasor current ratio.

$$\frac{I_1}{I_2} = e^{\gamma}$$

Where γ is a complex number defined by

$$\gamma = \alpha + j\beta$$

Hence $\dfrac{I_1}{I_2} = e^{\gamma} = e^{\alpha + j\beta}$

If $\dfrac{I_1}{I_2} = A\angle\beta$

$$A = \left|\frac{I_1}{I_2}\right| = e^{\alpha} \qquad \beta = e^{j\beta}$$

With Z_0 termination, it is also truc,

$$\left|\frac{V_1}{V_2}\right| = e^{\gamma}$$

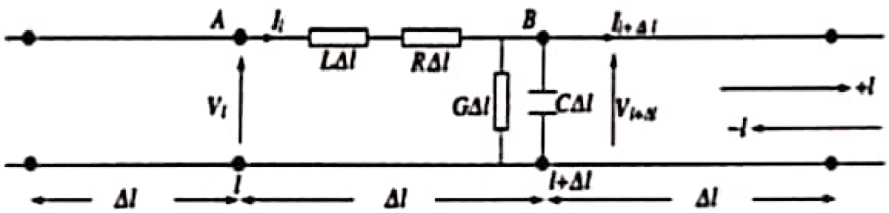
- The term γ has been given the name propagation constant
- α=attenuation constant, it determines the magnitude ratio between input and output quantities.
- It is the attenuation produced in passing the network.
- Units of attenuation is nepers

β= phase constant. It determines the phase angle between input and output quantities.

If phase shift occurs, it indicates the propagation of signal through the network.

The unit of phase shift is radians.

Transmission line equations



$$Z = R\Delta l + jwL\Delta l$$

$$Y = G\Delta l + jwC\Delta l$$

- Line equations are derived by assuming large number of short segments.
- The total series impedance of the line segment is Z
- The total parallel line admittance of the line segment is Y
- By applying Kirchhoff's voltage and current law

$$V(l) = I(l)[R\Delta l + jwL\Delta l] + V(l+\Delta l) \Rightarrow \frac{dV(l)}{dl} = -I(l)[R + jwL\Delta l]$$

$$I(l) = I(l+\Delta l) + V(l+\Delta l)[G + jwC\Delta l] \Rightarrow \frac{dI(l)}{dl} = -V(l+\Delta l)[R + jwL\Delta l]$$

Transmission line equations cont...

- By using the taylor expansion for *V(l+Δl)* about *l*. *V(l+Δl)=V(l)*. $\frac{dI(l)}{dl} = -V(l)[R + jwL\Delta l]$
- By combining the two differential equations

$$\frac{d^2V}{dl^2} - \gamma^2 V = 0$$

$$\frac{d^2I}{dl^2} - \gamma^2 I = 0$$

$$\gamma = \alpha + j\beta = \sqrt{(R + jwL)(G + jwC)}$$

- The solution for voltage and current is

$$V(l) = V^+ e^{-\gamma l} + V^- e^{+\gamma l}$$

$$I(l) = I^+ e^{-\gamma l} + I^- e^{+\gamma l}$$

Types of transmission lines

1. Lossless transmission line ($\alpha=0$).
2. Infinite long transmission line (No reflection from load).
3. Distortion-less transmission line (α,Z independent of frequency)
4. Low resistive transmission line (R=0).

1. Lossless transmission line

- R=0 and G=0 we leads to α=0.
- Line is made of pure conductor.
- Practically not existing only approximated line exist.
- The field components propagate along line with speed dictated by L and C.

$$\gamma = j\beta = jw\sqrt{LC}[rad/m]$$

$$Z_0 = \sqrt{\frac{L}{C}}[\Omega]$$

$$\lambda = \frac{2\pi}{w\sqrt{LC}}[m]$$

$$v_p = \frac{1}{\sqrt{LC}}[m/s]$$

3. Distortion less transmission line

- This is line whose impact on propagation wave is independent of frequncy.
- General lossy line with attenuation constant, phase velocity and characteristic impedance independent of frequency.
- For a distortion less the line parameters must be designed so that R/L=G/C.

$$\gamma = R\sqrt{\frac{C}{L}} + jw\sqrt{LC}$$

$$\alpha = R\sqrt{\frac{C}{L}} = \sqrt{RG}$$

$$\beta = w\sqrt{LC}$$

$$v_p = \frac{w}{\beta} = \frac{1}{\sqrt{LC}}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$$

4. Low resistance transmission line

- R=0.
- These lines are made of pure conductors.
- The conducting nature of the line guides the wave but all the propagation parameters are effected by dielectric alone.
- These equations can holds for any line therefore by knowing one parameters remaining can be measured.

$$\gamma = jw\sqrt{LC}\sqrt{1+\frac{G}{jwC}}$$

$$Z_0 = \sqrt{\frac{jwL}{G+jwC}}$$

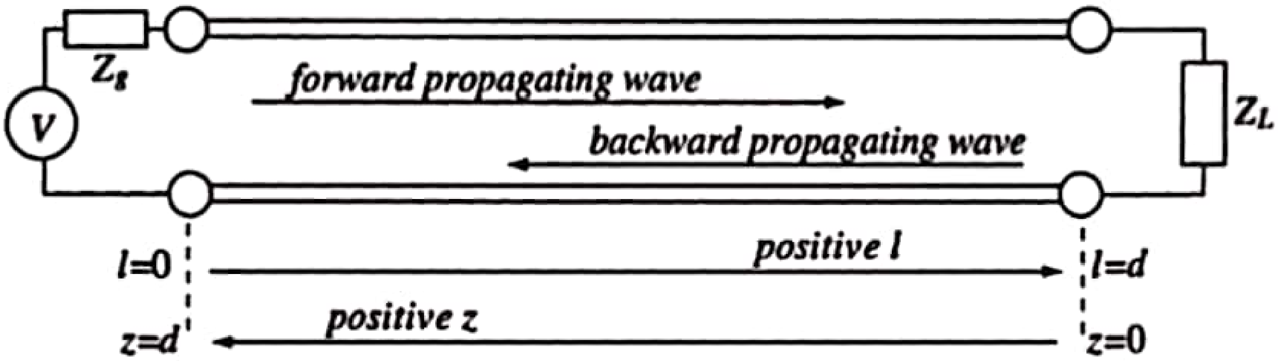
$$LC = \mu\varepsilon, \frac{G}{C} = \frac{\sigma}{\varepsilon}$$

Finite Transmission Lines

- A finite line connected between the generator and load as shown in figure.
- For the analysis of line a reference point is needed on the line.
- The analysis till now are in terms of *l*, which is valid if generator is reference point and all analysis can be modified to *z* by considering load as a reference point.

$$V(l) = V^+ e^{-\gamma l} + V^- e^{+\gamma l} \qquad V(z) = V^+ e^{\gamma z} + V^- e^{-\gamma z}$$

$$I(l) = I^+ e^{-\gamma l} + I^- e^{+\gamma l} \qquad I(z) = I^+ e^{\gamma z} + I^- e^{-\gamma z}$$



1. The Load Reflection Coefficient

- Load Reflection coefficient is ratio of reflected voltage (back propagated) to the incident voltage (forward propagated).
- Reflection coefficient can be calculated using characteristic impedance and load impedance. Non-zero reflection coefficient represents mismatch of load impedance with line impedance.

$$Z_0 = \frac{V^+}{I^+} = -\frac{V^-}{I^-}$$

$$Z_L = \frac{V_L}{I_L} = \frac{V(0)}{I(0)}$$

$$Z_L = Z_0 \frac{V^+ + V^-}{V^+ - V^-}$$

$$\Gamma_L = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = |\Gamma_L| e^{j\theta_r}$$

Load Reflection coefficient is a complex number and it represents reflection coefficient at the load only

3. The Lossless, Terminated Transmission Line cont...

- The ratio between the maximum and minimum voltage (or current) is called **standing wave ratio.**

$$SWR = \frac{V_{\max}}{V_{\min}} = \frac{I_{\max}}{I_{\min}} = \frac{1+|\Gamma(z)|}{1-|\Gamma(z)|}$$

$$|\Gamma(z)| = \frac{SWR-1}{SWR+1}$$

$$V_{\max} = |V^+|(1+|\Gamma(z)|) = |V^+|\left(\frac{2SWR}{SWR+1}\right)$$

$$V_{\min} = |V^+|(1-|\Gamma(z)|) = |V^+|\left(\frac{2}{SWR+1}\right)$$

3. The Lossless, Terminated Transmission Line cont…

- A number of particular loads are as follow:
  1. Matched load: $Z_L = Z_0; \Gamma_L = 0$
  2. Short-circuited load: $Z_L = 0; \Gamma_L = -1$
  3. Open circuit load: $Z_L = \infty; \Gamma_L = +1$
  4. Resistive load: $Z_L = R_L + j0; -1 < \Gamma_L < +1$

4. Lossless matched transmission line

- The line voltage and current have only forward propagating wave.
- No standing wave in the line and all power on line transferred to load.

$$Z_L = Z_0$$

$$\Gamma_L = 0$$

$$Z(z) = Z_0$$

$$V(z) = V^+ e^{j\beta z}$$

$$I(z) = \frac{V^+}{Z_0} e^{j\beta z}$$

5. Lossless shorted transmission line

- The line impedance is purely imaginary and varies from –infinite to infinite.
- Load reflection coefficient is -1.
- Standing wave ratio is infinite.

$$Z_L = 0$$

$$\Gamma_L = -1$$

$$SWR = \infty$$

$$Z(z) = jZ_0 \tan(\beta z)$$

$$V(z) = V^+ e^{j\beta z}(1 - e^{-j2\beta z})$$

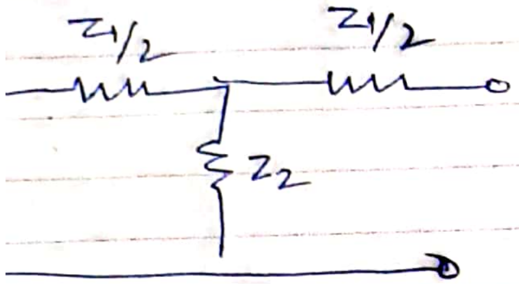
$$I(z) = \frac{V^+}{Z_0} e^{j\beta z}(1 + e^{-j2\beta z})$$

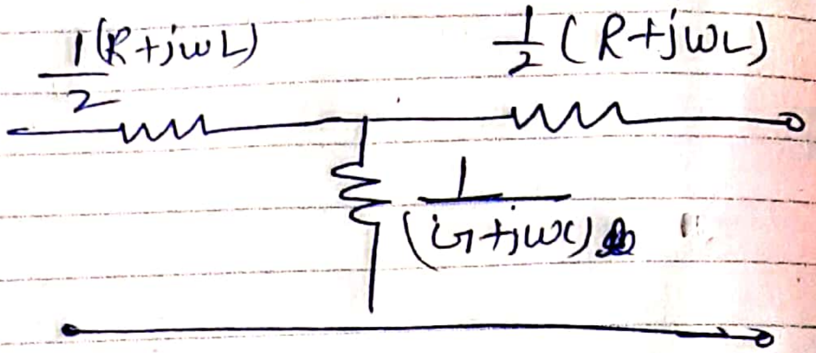
$$V_L = 0, I_L = \frac{2V^+}{Z_0}$$

$Z = R + j\omega L$ ohm/km
$Y = G + j\omega C$ mho/km

The infinite Line –

Relationship Between Primary & Secondary Const





for small section, Rl, Ll, Cl & Gl

for T – section

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$Z_1 = (R + j\omega L)l \quad \& \quad Z_2 = \frac{1}{(G + j\omega C)l}$$

$$Z_0 = \sqrt{\frac{(R+j\omega L)^2 l^2}{4} + \frac{(R+j\omega L)l}{(G+j\omega C)l}}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{Z}{Y}}$$

↳ Characteristic Impedance.

<u>Propagation Constant</u> –

$$\frac{I_S}{I_R} = e^{\gamma}$$

$\gamma = \alpha + j\beta$ → Phase Constant

α ↓ Attenuation Constant

$$\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

(Squaring Both Side)

$$\gamma^2 = ZY = \left(\sqrt{(R+j\omega L)(G+j\omega C)}\right)^2$$

$$(\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2 + j^2\beta^2 + 2\alpha j\beta = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2 - \beta^2 + 2\alpha j\beta = (R+j\omega L)(G+j\omega C)$$

Comparing Real & Imaginary term.